

Probing Models of Quantum Space–Time Foam

John Ellis ¹

CERN, Theory Division, Geneva 23 CH-1211, Switzerland

N.E. Mavromatos ²

Wheatstone Physics Laboratory, King’s College London, Strand, London WC2R 2LS, UK

D.V. Nanopoulos ³

Department of Physics, Texas A & M University, College Station, TX 77843-4242, USA,
Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA, and Academy of Athens, Chair of Theoretical Physics, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece.

Abstract. We review the possibility that quantum fluctuations in the structure of space-time at the Planck scale might be subject to experimental probes. We study the effects of space-time foam in an approach inspired by string theory, in which solitonic D -brane excitations are taken into account when considering the ground state. We model the properties of this medium by analyzing the recoil of a D particle which is induced by the scattering of a closed-string state. We find that this recoil causes an energy-dependent perturbation of the background metric, which in turn induces an energy-dependent refractive index *in vacuo*, and stochastic fluctuations of the light cone. We show how distant astrophysical sources such as Gamma-Ray Bursters (GRBs) may be used to test this possibility, making an illustrative analysis of GRBs whose redshifts have been measured. Within this framework, we also discuss the propagation of massive particles and the possible appearance of cosmological vacuum energy that relaxes towards zero. We also discuss D -brane recoil in models with ‘large’ extra dimensions.

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¹ E-mail: John.Ellis@cern.ch

² E-mail: Nikolaos.Mavromatos@cern.ch

³ E-mail: dimitri@soda.physics.tamu.edu

1. Introduction and Orientation

The concept of space time foam is an old one, first suggested by J.A. Wheeler [1], which has subsequently reappeared in various forms: see, for example [2, 3, 4, 5, 6, 7, 8]. The basic intuition is that quantum-gravitational fluctuations in the fabric of space-time cause the vacuum to behave like a stochastic medium. Most physicists who have studied the problem would surely agree that quantum-gravitational interactions must alter dramatically our classical perception of the space-time continuum when one attains Planckian energy scales $E \sim M_P \simeq 10^{19}$ GeV or Planckian distance scales $\ell \sim \ell_P \simeq 10^{-33}$ cm, which are the scales where gravitational interactions are expected to become strong. At issue are the following questions: how may classical space-time be altered at these scales? and is there any way of testing these possibilities?

At first sight, it might seem impossible to test such a suggestion within the foreseeable future, given the restrictions on the energies attainable with particle accelerators, and their corresponding limitations as microscopes. However, there are many instances in which new physics has revealed itself as a novel phenomenon far below its intrinsic mass scale M , a prime example being the weak interaction. In general, new-physics are suppressed by some inverse power of the heavy mass scale M , e.g., weak-interaction amplitudes are suppressed by $1/M_W^2$. However, some new-physics effects may be suppressed by just one power of the heavy mass scale, e.g., proton-decay amplitudes in some supersymmetric GUTs are $\propto 1/M_{GUT}$. Although gravitational amplitudes are generally suppressed by $1/M_P^2$, one should be open to the possibility that some quantum-gravitational effects might be suppressed simply by $1/M_P$. Moreover, there are many suggestions nowadays that M_P might not be a fundamental mass scale, and the quantum-gravitational effects might appear at some much lower scale related to the size(s) of one or more extra dimensions [9].

It has sometimes been suggested that Lorentz invariance might require quantum-gravitational effects to be suppressed by at least $\mathcal{O}[(E/M_P)^2]$, where E is a typical low-energy scale. However, Lorentz invariance is a casualty of many approaches to quantum gravity, and it is not clear how the very concept of space-time foam could be formulated Lorentz-invariantly. For example, many approaches to the physics of very small distances suggest that the classical space-time continuum may no longer exist, but might instead be replaced by a cellular structure.

String theory, plausibly in its current formulation as M theory, is at present the best (only?) candidate for a true quantum theory of gravity, so it is natural to ask what guidance it may offer us into the possible observability of quantum-gravitational effects. A first tool for this task was provided by two-dimensional string models [10], but a much more powerful tool has now been provided by D (irichlet) branes [11]. In this talk we review one particular D -brane approach to the modelling of space-time foam [12]. A characteristic feature of this approach is a treatment of D -brane recoil effects, in an attempt to incorporate the back-reaction of propagating particles on the ambient metric. This leads to the sacrifice of Lorentz invariance at the Planck scale, and suggests that space-time-foam effects arise already at the order $\mathcal{O}(E/M_P)$, in which case they might well be observable.

We have argued in the past that such minimally-suppressed quantum-

gravitational effects could be probed in the neutral-kaon system [3], which is well known to be a sensitive laboratory for testing quantum mechanics and fundamental symmetries. In this talk, we focus more on the possibility that the propagation of other particles such as photons or neutrinos might be affected in a way that could be testable in the foreseeable future, for instance through astrophysical observations of pulsed sources such as γ -ray bursters (GRBs), active galactic nuclei (AGNs) or pulsars [5]. Our basic suggestion is that space-time foam may act as a non-trivial optical medium with, e.g., an energy- or frequency-dependent refractive index, through which the propagation of energetic particles might be slowed down so that they travel at less than the speed of light c : $\delta c/c \sim -E/M_P$. A secondary effect might be a stochastic spread in the velocities of particles with identical energies.

We describe in Section 2 below the motivations for such suggestions, starting from a (relatively) simplified non-technical description of the string-inspired prototype model of space-time foam based on one particular treatment of D branes [12, 13]. This model involves naturally the breaking of Lorentz covariance, as a sort of spontaneous breaking. The basic idea may be summarized as follows. In the modern view of string theory, D particles must be included in the consistent formulation of the ground-state vacuum configuration [11]⁴. We consider a closed-string state propagating in a $(D+1)$ -dimensional space-time, which impacts a very massive D (irichlet) particle embedded in this space-time. We argue that the scattering of the closed-string state on the D particle induces recoil of the latter, which distorts the surrounding space-time in a stochastic manner. From the point of view of the closed-string particle and any low-energy spectators, this is a non-equilibrium process, in which information is ‘lost’ during the recoil, being carried by recoiling D -brane degrees of freedom that are inaccessible to a low-energy observer. Thus, although the entire process is consistent with quantum-mechanical unitarity, the low-energy effective theory is characterized by information loss and entropy production. From a string-theory point of view, the loss of information is encoded in a deviation from conformal invariance of the relevant world-sheet σ model that describes the recoil, that is compensated by the introduction of a Liouville field [14], which in turn is identified with the target time in the approach [10] adopted here.

The reader who wishes may skip the next section and proceed directly to Section 3, where we discuss the possible phenomenology of this approach to space-time foam. We discuss the appearance of an energy-dependent refractive index *in vacuo* and stochastic fluctuations in the velocities of photons of the same energy. We then discuss the corresponding modification of Maxwell’s equations, and their consequences for the propagation of photon pulses. Subsequently, we make a sample analysis of data from distant astrophysical sources, using GRBs with measured redshifts. Section 4 contains a discussion of the propagation of massive particles, brief comments on the possible appearance of cosmological vacuum energy within this approach, and an exploration of D -brane recoil effects in models with ‘large’

⁴ For stability reasons, the ground state must be supersymmetric. However, supersymmetry is not essential for our analysis, and we usually do not mention it explicitly until some comments near the end of the talk.

extra dimensions.

2. D -Particle Recoil Model for Space-Time Foam

We recall that D branes are solitons in string theory [11], of mass $m\sqrt{\alpha'} \sim 1/g_s$, where $\alpha' \equiv \ell_s^2$ is the Regge slope (string scale), and g_s is the string coupling, which is assumed throughout this work to be weak. We consider the situation depicted in Fig. 1, namely the scattering of a closed-string state (which might represent a photon) on a D brane embedded in a $D + 1$ -dimensional space-time. We assume that the D brane is so heavy that a non-relativistic approximation is sufficient. As already mentioned, we assume that the vacuum is (approximately) supersymmetric, and that the vacuum energy vanishes (approximately). We believe that an instructive analogy may be drawn between the closed-string/ D -particle system and a system of valence electrons moving freely through an ion lattice in a solid. Ion-lattice vibrations (c.f., the recoil of D particles) are well-known to induce effective phonon interactions, which bind the electrons resulting in BCS superconductivity for the ground state of this many-body system. In this example, the physical excitations above this ground state are quasiparticles, which are not the ordinary electrons, but are ‘dressed’ by their interactions with the excitations in the background medium, affecting their properties. In the language of relativistic cosmology, the D branes can be regarded as providing a ‘material reference system’ (MRS). At a fundamental level, the non-rigid recoil of D branes implies that the vacuum becomes a non-trivial medium.

The study of D -brane dynamics has been made possible by Polchinski’s realization [11] that such solitonic string backgrounds can be described in a conformally-invariant way, in terms of world sheets with boundaries (thus incorporating open strings), on which Dirichlet boundary conditions for the collective target-space coordinates of the soliton are imposed. The remarkably simple construction of Polchinski [11] opens the way to a σ -model description of such D -brane excitations, in which the critical world-sheet string action is perturbed using appropriate boundary terms. We recall the form of the world-sheet boundary operators describing the excitation of a D brane [11]:

$$\mathcal{V}_D = \int_{\partial\Sigma} (y_i \partial_n X^i + u_i X^0 \partial_n X^i) \quad (1)$$

where n denotes the normal derivative on the boundary of the world sheet $\partial\Sigma$, which has at tree level the topology of a disk of size L , and the X^i , $i = 1, \dots$ denote the collective excitations of the D brane, which satisfy Dirichlet boundary conditions on the world-sheet boundary:

$$X^i(\text{boundary}) = 0, \quad i = 1, \dots, \quad (2)$$

whilst X^0 is the target time variable which satisfies standard Neumann boundary conditions: $\partial_n X^0(\text{boundary}) = 0$. For simplicity, we consider later in more detail the case of a 0 brane, or D particle ⁵, with the quantity u_i in (1) denoting its

⁵ See [15] for a more general discussion of D -brane/ D -brane scattering.

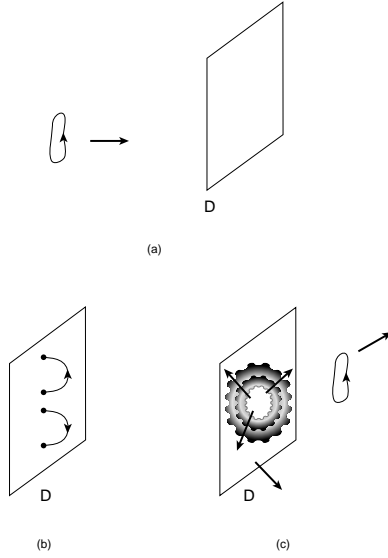


Figure 1. *The scattering of a low-energy closed-string state on a D brane: (a) the asymptotic past, (b) the time of impact ($X^0 = 0$), with trapping of the string state on the D -brane surface by a split into two open-string excitations, and (c) the asymptotic future, after two open strings recombine to emit a closed-string state, while the D brane recoils with finite velocity and its internal state fluctuates.*

velocity, and y_i its initial position. In this case, the operators (1) describe shifts and motion of the 0 brane, and so can be thought of as generating the action of the Poincaré group on the D particle, with the y_i parametrizing translations and the u_i parametrizing boosts. In the general D -brane case, these represent translations and boosts acting on the surface \mathcal{S} of the D brane.

As a first step towards the quantum theory of the scattering of a closed-string state on such a D brane, we consider its motion towards the (initially fixed) surface \mathcal{S} shown in Fig. 1(a), the latter viewed as a string soliton background. In the general D -brane case, the surface \mathcal{S} divides the target space-time into two regions. The closed-string state is initially far away from the surface of the brane. At a certain moment, say $X^0 = 0$, the incoming closed-string state finds itself lying partly outside and partly inside the D -brane surface. There are then two possibilities to be considered: it may be either absorbed Fig. 1(b) or rescattered as in Fig. 1(c). In general, quantum scattering on the D brane excites an open-string state on its surface, which in the scattering case of Fig. 1(c) also emits another closed-string state. The quantum excitation and emission processes are both described by closed-to-open string amplitudes, which are non-zero in a world-sheet theory with boundaries. The open-string states are excitations on the D -brane collective-coordinate surface. As was shown in [16], such processes can be described in terms of data of the bulk theory. As we showed in [12], tracing over such excitations results in a quantum modification of the Hawking-Bekenstein area law for the entropy,

which has been shown to hold in tree-level treatments of D branes.

This modification is due to the essence of our approach, which reflects the basic property of General Relativity that *there are no rigid bodies*, and hence the recoil fluctuations of the brane and their back-reaction on the surrounding space-time medium cannot be neglected.

A correct world-sheet treatment of recoil requires [17] an operator with non-zero matrix elements between different D - (in our case 0-)brane states. This can be achieved in the impulse approximation by introducing a Heaviside-function factor $\Theta(X^0)$ into the second operator in (1), so as to describe a 0 brane that starts moving at time $X^0 = 0$, when the initial position of the 0 brane at $X^0 = 0$ is given by the y_i . To determine the precise form of the recoil operator in our case, we observe that the leading quantum correction to the scattering of a closed string on a D brane is given by an annulus, as shown in Fig. 2(a). This is divergent in the limit where the annulus is pinched, as shown in Fig. 2(b). The weakly-coupled string limit: $g_s \rightarrow 0$, that we consider here, corresponds in leading order to a one-open-string-loop (annulus) analysis for a semi-classical (heavy) 0 brane (D particle), since the mass of the latter is $M_D \propto 1/g_s$ in natural string units.

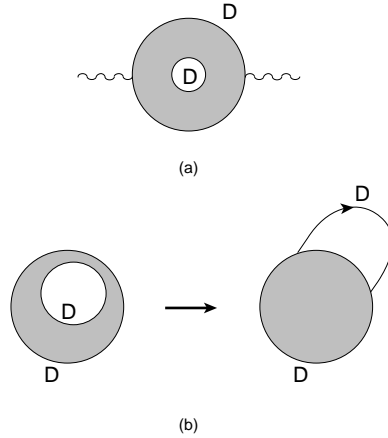


Figure 2. (a) : World-sheet annulus diagram for the leading quantum correction to the propagation of a string state in a D -brane background, and (b) the pinched annulus configuration which is the dominant divergent contribution to the quantum recoil.

In the case of a D -brane string soliton, its recoil after interaction with a closed-string state [17] is characterized by a pair of logarithmic operators [18]:

$$C_\epsilon \sim \epsilon \Theta_\epsilon(t), \quad D_\epsilon \sim t \Theta_\epsilon(t) \quad (3)$$

defined on the boundary $\partial\Sigma$ of the string world sheet. The operators (3) act as deformations of the conformal field theory on the world sheet: $u_i \int_{\partial\Sigma} \partial_n X^i D_\epsilon$ describes the shift of the D brane induced by the scattering, where u_i is its recoil velocity, and $y_i \int_{\partial\Sigma} \partial_n X^i C_\epsilon$ describes quantum fluctuations in the initial position

y_i of the D particle. It has been shown [19] that energy-momentum is conserved during the recoil process:

$$u_i = k_1 - k_2, \quad (4)$$

where $k_1(k_2)$ is the momentum of the propagating closed-string state before (after) the recoil, as a result of the summation over world-sheet genera. Thus the result (4) is an exact result, as far as world-sheet perturbation theory is concerned. We also note that $u_i = g_s P_i$, where P_i is the momentum and g_s is the string coupling, which is assumed here to be weak enough to ensure that D branes are very massive, with mass $M_D = 1/(\ell_s g_s)$, where ℓ_s is the string length.

The correct specification of the logarithmic pair (3) entails a regulating parameter $\epsilon \rightarrow 0^+$, which appears inside the $\Theta_\epsilon(t)$ operator: $\Theta_\epsilon(t) = \int \frac{d\omega}{2\pi} \frac{1}{\omega - i\epsilon} e^{i\omega t}$. In order to realize the logarithmic algebra between the operators C and D , one takes [17]:

$$\epsilon^{-2} \sim \text{Log} \Lambda / a \equiv \alpha \quad (5)$$

where Λ (a) are infrared (ultraviolet) world-sheet cut-offs. The pertinent two-point functions then have the following form [17]:

$$\begin{aligned} \langle C_\epsilon(z) C_\epsilon(0) \rangle &\sim 0 + O[\epsilon^2] \\ \langle C_\epsilon(z) D_\epsilon(0) \rangle &\sim 1 \\ \langle D_\epsilon(z) D_\epsilon(0) \rangle &\sim \frac{1}{\epsilon^2} - 2\eta \log |z/L|^2 \end{aligned} \quad (6)$$

up to an overall normalization factor, which is the logarithmic algebra [18] in the limit $\epsilon \rightarrow 0^+$, modulo the leading divergence in the $\langle D_\epsilon D_\epsilon \rangle$ recoil correlator. This leading divergent term will be important for our subsequent analysis.

The recoil deformations of the $D0$ brane (3) are relevant deformations, in the sense of conformal field theory, with anomalous dimension $-\epsilon^2/2$. However [19], the velocity operator D_ϵ (3) becomes exactly marginal, in a world-sheet renormalization-group sense, when it is divided by ϵ , in which case the recoil velocity is renormalized [19]:

$$u_i \rightarrow \bar{u}_i \equiv u_i / \epsilon \quad (7)$$

and becomes exactly marginal, playing the rôle of the physical velocity of the recoiling D particle.

Although such a renormalization is compatible with global world-sheet scaling, local world-sheet scale (conformal) symmetry is broken by the non-marginal character of the deformations (3), and restoration of conformal invariance requires Liouville dressing [14]. To determine the effect of such dressing on the space-time geometry, it is essential to write [12, 13] the boundary recoil deformation as a bulk world-sheet deformation

$$\int_{\partial\Sigma} d\tau \bar{u}_i X^0 \Theta_\epsilon(X^0) \partial_n X^i = \int_{\Sigma} d^2\sigma \partial_\alpha ([\bar{u}_i X^0] \Theta_\epsilon(X^0) \partial_\alpha X^i) \quad (8)$$

where the \bar{u}_i denote the renormalized recoil couplings (7), in the sense discussed in [19]. As we have already mentioned, the couplings (8) are marginal on a flat world sheet, and become marginal on a curved world sheet if one dresses [14] the

bulk integrand with a factor $e^{\alpha_i \phi}$, where ϕ is the Liouville field and α_i is the gravitational conformal dimension. This is related to the flat-world-sheet anomalous dimension $-\epsilon^2/2$ of the recoil operator, viewed as a bulk world-sheet deformation, as follows [14]:

$$\alpha_i = -\frac{Q_b}{2} + \sqrt{\frac{Q_b^2}{4} + \frac{\epsilon^2}{2}} \quad (9)$$

where Q_b is the central-charge deficit of the bulk world-sheet theory. In the recoil problem at hand, as discussed in [20], $Q_b^2 \sim \epsilon^4/g_s^2$ for weak deformations. This yields $\alpha_i \sim -\epsilon$ to leading order in perturbation theory in ϵ , to which we restrict ourselves here.

We next remark that, as the analysis of [12] indicates, the X^0 -dependent operator $\Theta_\epsilon(X^0)$ scales as follows with ϵ for $X^0 > 0$: $\Theta_\epsilon(X^0) \sim e^{-\epsilon X^0} \Theta(X^0)$, where $\Theta(X^0)$ is a Heaviside step function without any field content, evaluated in the limit $\epsilon \rightarrow 0^+$. The bulk deformations, therefore, yield the following σ -model terms:

$$\frac{1}{4\pi\ell_s^2} \epsilon \sum_{I=m+1}^{D-1} g_{Ii} X^I e^{\epsilon(\phi_{(0)} - X_{(0)}^I)} \Theta(X_{(0)}^I) \int_{\Sigma} \partial^\alpha X^I \partial^\alpha y_i \quad (10)$$

where the subscripts (0) denote world-sheet zero modes.

When we interpret the Liouville zero mode $\phi_{(0)}$ as target time, $\phi_{(0)} \equiv X^0 = t$, the deformations (10) yield space-time metric deformations in a σ -model sense, which were interpreted in [12] as expressing the distortion of the space-time surrounding the recoiling D -brane soliton. For clarity, we now drop the subscripts (0) for the rest of this paper. The resulting space-time distortion is then described by the metric elements:

$$G_{ij} = \delta_{ij}, G_{00} = -1, G_{0i} = \epsilon(\epsilon y_i + \epsilon \bar{u}_i t) \Theta_\epsilon(t), \quad i = 1, \dots, D-1 \quad (11)$$

where the suffix 0 denotes temporal (Liouville) components.

The presence of $\Theta(t)$ functions in (11) implies that the induced space-time is piecewise continuous⁶, with a singularity in the Riemann curvature scalar as $t \rightarrow 0$:

$$\begin{aligned} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} &= 4(D-1)\epsilon^4 [\delta_\epsilon(t)]^2 + \mathcal{O}(\epsilon^6) \\ R_{\mu\nu} R^{\mu\nu} &= D(D-1)\epsilon^4 [\delta_\epsilon(t)]^2 + \mathcal{O}(\epsilon^6) \\ R^2 &= 4(D-1)^2 \epsilon^4 [\delta_\epsilon(t)]^2 + \mathcal{O}(\epsilon^6) \end{aligned} \quad (12)$$

where $\delta_\epsilon(t)$ is the appropriate derivative of $\Theta_\epsilon(t)$ [20]. The reader should not be alarmed by the appearance of the $[\delta_\epsilon(t)]^2$ factors, which do not make ϵ -dependent divergent contributions to the physical quantities of interest to us, such as the integrated central-charge deficit Q , etc., even in the limit $\epsilon \rightarrow 0^+$. In our regularization, it can be easily shown that in this limit $\Theta_\epsilon(0) \rightarrow \pi$ whilst $\delta_\epsilon(0)$ becomes formally a linearly-divergent integral that is independent of ϵ .

⁶ This has important implications for non-thermal particle production and decoherence for a spectator low-energy field theory in such space-times, as discussed in [20, 12].

We note that the derivation of the space-time discussed above was essentially non-relativistic, since we worked in the approximation of a very heavy D brane. This is the reason why the singularity in the geometry (12) is space-like, and why the consequent change in the quantum state also occurs on a space-like surface. This apparent non-causality is merely an artefact of our approximation. We expect that, in a fully relativistic D -brane approach, the space-time singularity would travel along a light-cone, and that the quantum state would also change causally. However, here we are interested in the difference between early- and late-time quantum states, and this apparent non-causal behaviour is irrelevant.

As shown in [20], the metric (11) can be derived using the equations of motion for a suitable effective action S induced by quantum D -brane effects:

$$S = \int d^D x \sqrt{-G} \alpha' e^{-2\Phi} \hat{R}_{GB}^2 \quad (13)$$

where $\Phi = Qt$ is a linear dilaton field [21], with Q the appropriate central charge deficit of the world-sheet (non)conformal theory associated with the metric deformation (11), which was calculated in [20], and \hat{R}_{GB}^2 is the ghost-free Gauss-Bonnet quadratic combination of the Riemann tensor, Ricci tensor and curvature scalar:

$$\hat{R}_{GB}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \quad (14)$$

This action should not be confused with the ordinary Einstein action describing space-time far away from the defect. Our point is rather to show that the target-space-time deformations induced by recoil of the D -particle are compatible with equations of motion obtained from an effective action, and hence can be considered as consistent string backgrounds on which ordinary low-energy matter propagates. The next Section will consider the physical consequences for such propagation.

Before that, however, it is essential to discuss in some detail renormalization-group rescaling on the world sheet, with the aim of establishing a connection between the target-time variable t and ϵ . To this end, we consider a finite-size world-sheet scale transformation

$$L \rightarrow L' = L e^t \quad (15)$$

where L is the size of the world sheet in units of the ultraviolet cut-off: this is the only type of dimensionless scale that makes physical sense for the open-string world sheet. The relation (5) between ϵ and L entails the following transformation of ϵ :

$$\epsilon^2 \rightarrow \epsilon'^2 = \frac{\epsilon^2}{1 + 4\eta\epsilon^2 t} \quad (16)$$

We deduce from the scale dependences of the correlation functions (6) that the corresponding transformations of C_ϵ and D_ϵ are:

$$\begin{aligned} D_\epsilon &\rightarrow D_{\epsilon'} = D_\epsilon - t C_\epsilon \\ C_\epsilon &\rightarrow C_{\epsilon'} = C_\epsilon \end{aligned} \quad (17)$$

We emphasize that this transformation law is unambiguous.

The corresponding transformation laws for the couplings y_i and u_i , which are conjugate to D_ϵ and C_ϵ , are

$$u_i \rightarrow u_i \quad , \quad y_i \rightarrow y_i + u_i t \quad (18)$$

These are consistent with the interpretations of u_i as the velocity after the scattering process and y_i as the spatial collective coordinates of the brane, if and only if the parameter ϵ^{-2} is identified with the target Minkowski time t for $t \gg 0$ after the collision:

$$\epsilon^{-2} \simeq t \quad (19)$$

We have assumed in this analysis that the velocity u_i is small, as is appropriate in the weak-coupling régime studied here. The D -brane σ -model formalism is completely relativistic, and we anticipate that a complete treatment beyond the one-loop order discussed here will incorporate correctly all relativistic effects, including Lorentz factors wherever appropriate.

In view of (19), one observes that for $t \gg 0$ the metric (11) becomes to leading order:

$$G_{ij} = \delta_{ij}, G_{00} = -1, G_{0i} \sim \bar{u}_i, \quad i = 1, \dots, D-1 \quad (20)$$

which is constant in space-time. However, the effective metric depends on the energy content of the low-energy particle that scattered on the D -particle, because of momentum conservation during the recoil process (4). This energy dependence is the primary deviation from Lorentz invariance induced by the D -particle recoil.

3. Phenomenological Implications for the Propagation of Photons

3.1. Refractive Index in Vacuo

We now proceed to discuss possible phenomenological consequences of the above phenomena, starting with the propagation of photons and relativistic particles. The above discussion of recoil suggests that the space-time background should be regarded as a non-trivial. Light propagating through media with non-trivial optical properties may exhibit a frequency-dependent refractive index, namely a variation in the light velocity with photon energy. Another possibility is a difference between the velocities of light with different polarizations, namely birefringence, and a third is a diffusive spread in the apparent velocity of light for light of fixed energy (frequency). Within the framework described in the previous Section, the first [5] and third [13] effects have been derived via a formal approach based on a Born-Infeld Lagrangian using D -brane technology ⁷. A different approach to light propagation has been taken in [23], where quantum-gravitational fluctuations in the light-cone have been calculated. Here we use this formalism together with the microscopic model background obtained in the previous Section to derive a non-trivial refractive index and a diffusive spread in the arrival times of photons of given frequency.

⁷ The possibility of birefringence has been raised [22] within a canonical approach to quantum gravity, but we do not pursue such a possibility here.

We first review briefly the analysis in [23], which considered gravitons in a squeezed coherent state, the natural result of quantum creation in the presence of black holes. Such gravitons induce quantum fluctuations in the space-time metric, in particular fluctuations in the light-cone [23], i.e., stochastic fluctuations in the velocity of light propagating through this ‘medium’. Following [23], we consider a flat background space-time with a linearized perturbation, corresponding to the invariant metric element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = dt^2 - d\vec{x}^2 + h_{\mu\nu}dx^\mu dx^\nu$. Let $2\sigma(x, x')$ be the squared geodesic separation for any pair of space-time points x and x' , and let $2\sigma_0(x, x')$ denote the corresponding quantity in a flat space-time background. In the case of small gravitational perturbations about the flat background, one may expand $\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \dots$, where σ_n denotes the n -th order term in an expansion in the gravitational perturbation $h_{\mu\nu}$. Then, as shown in [23], the root-mean-square (RMS) deviation from the classical propagation time Δt is related gauge-invariantly [23] to $\langle \sigma^2 \rangle$ by

$$\Delta t = \frac{\sqrt{\langle \sigma^2 \rangle - \langle \sigma_0^2 \rangle}}{L} \simeq \frac{\sqrt{\langle \sigma_1^2 \rangle}}{L} + \dots \quad (21)$$

where $L = |x' - x|$ is the distance between the source and the detector.

As commented earlier, one may expect Lorentz invariance to be broken in a generic theory of quantum gravity, and specifically in the recoil context discussed in the previous Section. In the context of string theory, violations of Lorentz invariance entail the exploration of non-critical string backgrounds, since Lorentz invariance is related to the conformal symmetry that is a property of critical strings. As we discussed in the previous Section, a general approach to the formulation of non-critical string theory involves introducing a Liouville field [14] as a conformal factor on the string world sheet, which has non-trivial dynamics and compensates the non-conformal behaviour of the string background, and we showed in the specific case of D branes that their recoil after interaction with a closed-string state produces a local distortion of the surrounding space-time (11).

Viewed as a perturbation about a flat target space-time, the metric (11) implies that the only non-zero components of $h_{\mu\nu}$ are:

$$h_{0i} = \epsilon^2 \bar{u}_i t \Theta_\epsilon(t) \quad (22)$$

in the case of D -brane recoil. We now consider light propagation along the x direction in the presence of a metric fluctuation h_{0x} (22) in flat space, along a null geodesic given by $(dt)^2 = (dx)^2 + 2h_{0x}dt dx$. For large times $t \sim \text{Log}\Lambda/a \sim \epsilon^{-2}$ [20], $h_{0x} \sim \bar{u}$, and thus we obtain

$$\frac{cdt}{dx} = \bar{u} + \sqrt{1 + \bar{u}^2} \sim 1 + \bar{u} + \mathcal{O}(\bar{u}^2) \quad (23)$$

where the recoil velocity \bar{u} is in the direction of the incoming light ray. Taking into account energy-momentum conservation in the recoil process, which has been derived in this formalism as mentioned previously, one has a typical order of magnitude $\bar{u}/c = \mathcal{O}(E/M_D c^2)$, where $M_D = g_s^{-1} M_s$ is the D -brane mass scale, with $M_s \equiv \ell_s^{-1}$. Hence (23) implies a subluminal energy-dependent velocity of light:

$$c(E)/c = 1 - \mathcal{O}(E/M_D c^2) \quad (24)$$

which corresponds to a *classical* refractive index. This appears because the metric perturbation (22) is energy-dependent, through its dependence on \bar{u} .

The subluminal velocity (24) induces a delay in the arrival of a photon of energy E propagating over a distance L of order:

$$(\Delta t)_r = \frac{L}{c} \mathcal{O} \left(\frac{E}{M_D c^2} \right) \quad (25)$$

This effect can be understood physically from the fact that the curvature of space-time induced by the recoil is \bar{u} - and hence energy-dependent. This affects the paths of photons in such a way that more energetic photons see more curvature, and thus are delayed with respect to low-energy ones.

As recalled above, the absence of superluminal light propagation was found previously via the formalism of the Born-Infeld lagrangian dynamics of D branes [19, 13]. Furthermore, the result (25) is in agreement with the analysis of [24, 5], which was based on a more abstract analysis of Liouville strings. It is encouraging that this result appears also in this more conventional general-relativity approach [23], in which the underlying physics is quite transparent.

3.2. Quantum Corrections and Stochastic Effects

As a preliminary to evaluating quantum effects [13] in the context of our simplified model, we now express σ_1^2 in terms of the two-point function of h_{0x} , considering again the null geodesic in the presence of the small metric perturbations (22). To first order in the recoil velocity, one has: $c(\Delta t) = \left(\Delta x - 2 \int_{x_1}^{x_2} dx h_{0x} \right)$, from which we find $2\sigma \simeq (\Delta t)^2 - (\Delta x)^2 + 2\epsilon^2 \bar{u} t \Theta_\epsilon(t) (\Delta x)^2$, implying that $\sigma_1 = \epsilon^2 \bar{u} t \Theta_\epsilon(t)$. One then has:

$$\langle \sigma_1^2 \rangle \sim L^2 \int_x^{x'} \int_x^{x'} dy \int dy' \langle h_{0x}(y, t) h_{0x}(y', t') \rangle \quad (26)$$

In the case of D -brane recoil, the computation of the quantum average $\langle \dots \rangle$ may be made in the Liouville-string approach described above. In this case, the quantum average $\langle \dots \rangle$ is replaced by a world-sheet correlator calculated with a world-sheet action deformed by (3). It is clear from (22) that the two-point metric correlator appearing in (26) is just the $\langle D_\epsilon D_\epsilon \rangle$ world-sheet recoil two-point function described in (6). Thus, at the classical tree level on the world sheet, one recovers the refractive index (25) from (26) by concentrating on the leading divergence in the $D_\epsilon D_\epsilon$ correlator (6), proportional to ϵ^{-2} .

To describe fully the quantum effects, one must sum over world-sheet genera. As discussed in [12, 19] such a procedure results in a canonical quantization of the world-sheet couplings, which, in the problem at hand, coincide with the target-space collective coordinates and momenta of the recoiling D brane. Thus, the quantum effects arising from summation over world-sheet genera result, in a first approximation, in $\langle \sigma_1^2 \rangle \sim L^4 (\Delta \bar{u})^2$, where $\Delta \bar{u}$ denotes the quantum uncertainty of the recoil velocity \bar{u} , which has been computed in [19].

The result of the summation over genera has been discussed in detail in [19], and is not repeated here: we only state the final results relevant for our purposes.

The leading contributions to the quantum fluctuations in the space \mathcal{M} of world-sheet couplings arise from pinched annulus diagrams in the summation over world-sheet genera [12, 19]. These consist of thin tubes of width $\delta \rightarrow 0$, which one may regard as wormholes attached to the world-sheet surface Σ . The attachment of each tube corresponds to the insertion of a bilocal pair of recoil vertex operators $V^i(s)V^j(s')$ on the boundary $\partial\Sigma$, with interaction strength g_s^2 , and computing the string propagator along the thin tubes. One effect of the dilute gas of world-sheet wormholes is to exponentiate the bilocal operator, leading to a change in the world-sheet action [12, 19]. This contribution can be cast into the form of a local action by rewriting it as a Gaussian functional integral over wormhole parameters ρ_i^{ab} , leading finally to [19]:

$$\sum_{\text{genera}} Z \simeq \int_{\mathcal{M}} D\rho \, e^{-\rho_i^{ab} G^{ij} \rho_j^{cd} / 2 |\epsilon|^2 \ell_s^2 g_s^2 \log \delta} \langle W[Y + \rho] \rangle_0 \quad (27)$$

where $\langle W[Y + \rho] \rangle_0$ denotes the partition function on a world sheet with the topology of a disc of a model deformed by the operators (3), with couplings shifted by ρ , and G^{ij} is the inverse of the Zamolodchikov metric [25] $\langle V_i V_j \rangle$ in string theory space, evaluated on the disc world sheet. The shifts in the effective couplings Y_i, u_i imply that they fluctuate statistically, in much the same way as wormholes and other topology changes in target space-times lead to the quantization of the couplings in conventional field theories [26].

In our case, we see from (27) that the effect of this resummation over pinched genera is to induce quantum fluctuations in the solitonic D -brane background, giving rise to a Gaussian statistical spread in the collective coordinates of the D brane, determined by G^{ij} for the logarithmic deformations (3). Note that, in such a formalism, one defines a renormalized string coupling $\bar{g}_s = g_s \epsilon^{-1}$, which plays the rôle of the physical string coupling in the problem [19]. To lowest non-trivial order in the string coupling \bar{g}_s and \bar{u}^2 , the world-sheet analysis of [19], using the methods of logarithmic conformal field theory [18], showed that the quantum fluctuations in \bar{u} induced by the summation over world-sheet topologies are given by;

$$(\Delta \bar{u})^2 = 4\bar{g}_s^2 \left[1 - \frac{285}{2} \bar{u}^2 \right], \quad (28)$$

if the D -brane foam corresponds to a minimum-uncertainty wave-packet [19], with the position fluctuations of the D branes being saturated: $\Delta Y_i \sim \bar{g}_s^{1/3} \ell_s$. The energy-independent first part of (28) can be absorbed into σ_0 , and hence does not contribute to the stochastic fluctuations (21) in the photon arrival time. The second part of the uncertainty (28), which depends on \bar{u} , and hence the energy of the photon, cannot be scaled away for all photons of different energies by a simple coordinate transformation. However, it is of higher (second) order in the small parameter \bar{u} . Thus the geodesic correction σ_1 , which is linear in the gravitational perturbation $h_{\mu\nu}$, contributes in leading order only to the classical refractive index discussed earlier.

We now repeat the analysis for the quadratic correction σ_2 in (21). This contains the normal-ordered coincidence limit of the two-point function of the D_ϵ operator, which has a leading-order term proportional to ϵ^{-2} times the c -number identity, as

seen in (6). Since we work in a subtraction scheme in which one-point functions of the deformations (3) vanish, this means that $\langle \sigma_1 \sigma_2 \rangle = 0$, so that the leading contribution comes from the sum over genera of $\langle \sigma_2^2 \rangle$, which yields fluctuations in the arrival time that are proportional to the uncertainty $\Delta(\bar{u}^2) \sim \bar{u} \Delta \bar{u}$. For a minimum-uncertainty state of D branes, we therefore find a contribution

$$|(\Delta t)_{obs}| \simeq \mathcal{O} \left(\bar{g}_s \frac{E}{M_D c^2} \right) \frac{L}{c} \quad (29)$$

to the RMS fluctuation in arrival times. As expected, the *quantum* effect (29) is suppressed by a power of the string coupling constant, when compared with the classical refractive index effect (25). The result (29) was also derived in [13] using the techniques of Liouville string theory, via the Born-Infeld lagrangian for the propagation of photons in the D -brane foam. Its derivation here using more conventional field-theoretic techniques adds robustness to the Liouville string theory perspective. It should be noted that the recoil-induced effect (29) is larger than the effects discussed in [23], which are related to metric perturbations associated with the squeezed coherent states relevant to particle creation in conventional local field theories.

3.3. Modification of Maxwell's Equations

We now consider [27] the effects of the recoil-induced space time (20), viewed as a ‘mean field solution’ of the D -brane-inspired quantum-gravity model, on the propagation of electromagnetic waves. Maxwell’s equations in the background metric (20) in empty space can be written as [28]:

$$\begin{aligned} \nabla \cdot B &= 0, & \nabla \times H - \frac{1}{c} \frac{\partial}{\partial t} D &= 0, \\ \nabla \cdot D &= 0, & \nabla \times E + \frac{1}{c} \frac{\partial}{\partial t} B &= 0, \end{aligned} \quad (30)$$

where

$$D = \frac{E}{\sqrt{h}} + H \times \mathcal{G}, \quad B = \frac{H}{\sqrt{h}} + \mathcal{G} \times E \quad (31)$$

Thus, there is a direct analogy with Maxwell’s equations in a medium with $1/\sqrt{h}$ playing the rôle of the electric and magnetic permeability. In our case [12], $h = 1$, so one has the same permeability as the classical vacuum. In the case of the constant metric perturbation (20), after some elementary vector algebra and, appropriate use of the modified Maxwell’s equations, the equations (30) read:

$$\begin{aligned} \nabla \cdot E + \bar{u} \cdot \frac{1}{c} \frac{\partial}{\partial t} E &= 0 \\ \nabla \times B - (1 - \bar{u}^2) \frac{1}{c} \frac{\partial}{\partial t} E + \bar{u} \times \frac{1}{c} \frac{\partial}{\partial t} B + (\bar{u} \cdot \nabla) E &= 0 \\ \nabla \cdot B &= 0 \\ \nabla \times E + \frac{1}{c} \frac{\partial}{\partial t} B &= 0 \end{aligned} \quad (32)$$

Dropping non-leading terms of order \bar{u}^2 from these equations, one obtains after some straightforward algebra the following modified wave equations for E and B :

$$\begin{aligned}\frac{1}{c^2} \frac{\partial^2}{\partial t^2} B - \nabla^2 B - 2(\bar{u} \cdot \nabla) \frac{1}{c} \frac{\partial}{\partial t} B &= 0 \\ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E - \nabla^2 E - 2(\bar{u} \cdot \nabla) \frac{1}{c} \frac{\partial}{\partial t} E &= 0\end{aligned}\quad (33)$$

If we consider one-dimensional motion along the x direction, we see that these equations admit wave solutions of the form

$$E_x = E_z = 0, \quad E_y(x, t) = E_0 e^{i(kx - \omega t)}, \quad B_x = B_y = 0, \quad B_z(x, t) = B_0 e^{i(kx - \omega t)}, \quad (34)$$

with the modified dispersion relation:

$$k^2 - \omega^2 - 2\bar{u}k\omega = 0 \quad (35)$$

Since the sign of \bar{u} is that of the momentum vector k along the x direction, the dispersion relation (35) corresponds to *subluminal* propagation with a refractive index:

$$c(E) = c(1 - \bar{u}) + \mathcal{O}(\bar{u}^2) \quad (36)$$

where we estimate that

$$\bar{u} = \mathcal{O}\left(\frac{E}{M_D c^2}\right) \quad (37)$$

with M_D the D -particle mass scale. This is in turn given by $M_D = g_s^{-1} M_s$ in a string model, where g_s is the string coupling and M_s is the string scale [13]. The relation (37) between \bar{u} and the photon energy has been shown [19] to follow from a rigorous world-sheet analysis of modular divergences in string theory, but the details need not concern us here. It merely expresses elementary energy-momentum conservation, as discussed earlier.

The refractive index effect (36) is a mean-field effect, which implies a delay in the arrival times of photons, relative to that of an idealized low-energy photon for which quantum-gravity effects can be ignored, of order:

$$\Delta t \sim \frac{L}{c} |\bar{u}| = \mathcal{O}\left(\frac{EL}{M_D c^3}\right) \quad (38)$$

As we have discussed above (see also [13]), one would also expect quantum fluctuations about the mean-field solution (38), corresponding in field theory to quantum fluctuations in the light cone that could be induced by higher-genus effects in a string approach. Such effects would result in stochastic fluctuations in the velocity of light which are of order

$$\delta c \sim 8g_s E / M_D c^2, \quad (39)$$

where g_s is the string coupling, which varies between $\mathcal{O}(1)$ and $\ll 1$ in different string models. Such an effect would motivate the following parametrization of any possible stochastic spread in photon arrival times:

$$(\delta \Delta t) = \frac{LE}{c\Lambda} \quad (40)$$

where the string approach suggests that $\Lambda \sim M_D c^2 / 8g_s$. We emphasize that, in contrast to the variation (36) in the refractive index - which refers to photons of different energy - the fluctuation (40) characterizes the statistical spread in the velocities of photons *of the same energy*. We recall that the stochastic effect (40) is suppressed, as compared to the refractive index mean field effect (37), by an extra power of g_s .

3.4. Consequences for the Propagation of Photon Pulses

We now discuss [27] the propagation of a pulse of photons through space-time foam (20). Several different types of astrophysical sources could be considered: for orientation, we will bear in mind GRBs, which typically emit photons in pulses with a combination of different wavelengths. Their sources are believed to be ultrarelativistic shocks with Lorentz factors $\gamma = \mathcal{O}(100)$ [29, 30]. We do not enter here into the details of the astrophysical modelling of such sources, which are unnecessary for our present exploratory study, though they may be essential for future more detailed probes of the constancy of the velocity of light. Instead, here we study a simple generalization of the previous discussion of monochromatic wave propagation, considering a wave packet of photons emitted with a Gaussian distribution in the light-cone variable $x - ct$. Since the distance over which the ultrarelativistic source moves during the emission is negligible compared with the distance between the source and the observer, we may represent the source equally well with a Gaussian distribution in x at the time $t = 0$. This is adequate to see how such a pulse would be modified at the observation point at a subsequent time t , because of the propagation through the space-time foam, as a result of the refractive index effect (35), (36). The phenomenon is similar to the motion of a wave packet in a conventional dispersive medium, as discussed extensively in the standard literature.

The Gaussian wavepacket may be expressed at $t = 0$ as the real part of

$$f(x) = A e^{-x^2/(\Delta x_0)^2} e^{ik_0 x} \quad (41)$$

with a modulation envelope that is symmetrical about the origin, where it has amplitude A . The quantity Δx_0 in (41) denotes the root mean square of the spatial spread of the energy distribution in the packet, which is proportional to $|f(x)|^2$, as is well known. If we assume a generic dispersion relation $\omega = \omega(k)$, a standard analysis using Fourier transforms shows that at time t the Gaussian wave-packet will have the form:

$$|f(x, t)|^2 = \frac{A^2}{(1 + \frac{\alpha^2 t^2}{(\Delta x_0)^4})^{1/2}} \exp\left\{-\frac{(x - c_g t)^2}{2(\Delta x_0)^2 [1 + \frac{\alpha^2 t^2}{(\Delta x_0)^4}]}\right\} \quad (42)$$

where $\alpha \equiv \frac{1}{2} (d^2 \omega / d^2 k)$, and $c_g \equiv d\omega / dk$ is the group velocity, i.e., the velocity with which the peak of the distribution moves in time.

We see immediately in (42) that the quadratic term α in the dispersion relation does not affect the motion of the peak, but only the spread of the Gaussian wave packet:

$$|\Delta x| = \Delta x_0 \left(1 + \frac{\alpha^2 t^2}{(\Delta x_0)^4}\right)^{1/2} \quad (43)$$

which thus increases with time. The quadratic term α also affects the amplitude of the wave packet: the latter decreases together with the increase in the spread (43), in such a way that the integral of $|f(x, t)|^2$ is constant.

In the case of the quantum-gravitational foam scenario of [13, 24], the dispersion relation assumes the following form for positive momentum k :

$$\begin{aligned} k &= \omega \left(1 + \frac{\omega}{M_D} \right) \text{ or } \omega = k \left(1 - \frac{k}{M_D} + \dots \right) \\ c_g &= (1 - \bar{u}) = 1 - \mathcal{O}(\omega/M_D), \\ \alpha &= -\frac{1}{M_D} + \dots \end{aligned} \quad (44)$$

where we denote by \dots the higher-order (e.g., quadratic) terms in $1/M_D$, which are subleading in this case. Thus the spread of the wave packet due to the non-trivial refractive-index effect described earlier is:

$$|\Delta x| = \Delta x_0 \left(1 + \frac{t^2}{M_D^2 (\Delta x_0)^4} \right)^{1/2} \quad (45)$$

We note that the spread due to the refractive index $\delta c/c \propto \omega$ is independent of the energy of the photon to leading order in $1/M_D$. We also note, therefore, that this effect is distinct from the stochastic propagation effect, which gives rise to a spread (40) in the wave-packet that depends on the photon energy ω . For astrophysical sources at cosmological distances with redshifts $z \simeq 1$, and with an initial Δx_0 of a few km, one finds that the correction (45) is negligible if the quantum-gravity scale M_D is of the order of 10^{19} GeV, namely of order $10^{-30} \Delta x_0$. The correction would become of order Δx_0 only if the latter is of order 10^{-3} m. Even if one allows M_D to be as low as the sensitivities shown in Table 1, this broadening effect is still negligible for all the sources there, being at most of order $10^{-22} \Delta x_0$. Therefore, in this particular model, the only broadening effect that needs to be considered is the stochastic quantum-gravitational effect on the refractive index that was introduced at the end of the previous subsection.

Table 1: Observational Sensitivities and Limits on M

Source	Distance	E	Δt	Sensitivity to M
GRB 920229 [5, 31]	3000 Mpc (?)	200 keV	10^{-2} s	0.6×10^{16} GeV (?)
GRB 980425 ^a	40 Mpc	1.8 MeV	10^{-3} s (?)	0.7×10^{16} GeV (?)
GRB 920925c ^a	40 Mpc (?)	200 TeV (?)	200 s	0.4×10^{19} GeV (?)
Mrk 421 [32]	100 Mpc	2 TeV	280 s	$> 7 \times 10^{16}$ GeV
Crab pulsar [33]	2.2 kpc	2 GeV	0.35 ms	$> 1.3 \times 10^{15}$ GeV
GRB 990123	5000 Mpc	4 MeV	1 s (?)	2×10^{15} GeV (?)

Table 1. *The mass-scale parameter M is defined by $\delta c/c = E/M$. The question marks in the Table indicate uncertain observational inputs. Hard limits are indicated by inequality signs.*

In the case of a different quantum-gravitational foam scenario with a quadratic refractive index: $\delta c/c \sim E^2$, the dispersion relation would take the following form:

$$\begin{aligned} k &= \omega \left(1 + \frac{\omega}{\tilde{M}} \right) \text{ or } \omega = k \left(1 - \left(\frac{k}{\tilde{M}} \right)^2 + \dots \right) \\ c_g &= (1 - \bar{u}) = 1 - \mathcal{O}(\omega^2/\tilde{M}^2), \\ \alpha &= -3\frac{\omega}{\tilde{M}} + \dots \end{aligned} \quad (46)$$

where \dots again denote subleading terms. In this case, the spread of the wave packet due to the non-trivial refractive index effect described above is:

$$|\Delta x| = \Delta x_0 \left(1 + \frac{9\omega^2 t^2}{\tilde{M}^4 (\Delta x_0)^4} \right)^{1/2} \quad (47)$$

Once again, if one takes into account the sensitivities shown in Table 1, the maximum spreading of the pulse is negligible for $\Delta x_0 \sim 10^{-3}$ m, namely at most $\sim 10^{-33} \Delta x_0$. Once again, one would need only to consider the possible stochastic quantum-gravitational effect on the refractive index. However, since a quadratic dependence is not favoured in our particular theoretical approach, we do not pursue it further in the rest of this paper.

3.5. Cosmological Expansion

We now discuss the implications of the cosmological expansion for the searches for a quantum-gravity induced refractive index (38) and the stochastic effect (40). We work within the general context of Friedmann-Robertson-Walker (FRW) metrics, as appropriate for standard homogeneous and isotropic cosmology [34]. We denote by R the FRW scale factor, adding a subscript 0 to denote the value at the present era, H_0 is the present Hubble expansion parameter, and the deceleration parameter q_0 is defined in terms of the curvature k of the FRW metric by $k = (2q_0 - 1)(H_0^2 R_0^2/c^2)$, i.e., $\Omega_0 = 2q_0$.

Motivated by inflation and the cosmic microwave background data, we assume a Universe with a critical density: $\Omega_0 = 1$, $k = 0$ and $q_0 = 1/2$. We also assume that the Universe is matter-dominated during all the epoch of interest. Then the scale factor $R(t)$ of the Universe expands as:

$$\frac{R(t)}{R_0} = \left(\frac{3H_0}{2} \right)^{2/3} t^{2/3} \quad (48)$$

and the current age of the Universe is

$$t_0 = \frac{2}{3H_0} \quad (49)$$

Clearly no time delay can be larger than this. The relation between redshift and scale factor is

$$R(t)/R_0 = 1/(1+z) \quad (50)$$

Substituting (50) into (48), we find the age of the Universe at any given redshift:

$$t = \left(\frac{2}{3H_0}\right) \frac{1}{(1+z)^{3/2}} = \frac{t_0}{(1+z)^{3/2}} \quad (51)$$

Hence, a photon (or other particle) emitted by an object at redshift z has travelled for a time

$$t_0 - t = \frac{2}{3H_0} \left(1 - \frac{1}{(1+z)^{3/2}}\right) \quad (52)$$

The corresponding differential relation between time and redshift is

$$dt = -\frac{1}{H_0} \frac{1}{(1+z)^{5/2}} dz \quad (53)$$

This means that during the corresponding infinitesimal time (redshift) interval, a particle with velocity u travels a distance

$$u dt = -\frac{1}{H_0} \frac{u}{(1+z)^{5/2}} dz. \quad (54)$$

Therefore, the total distance L travelled by such a particle since emission at redshift z is

$$L = \int_t^{t_0} u dt = \frac{1}{H_0} \int_0^z \frac{u(z)}{(1+z)^{5/2}} dz. \quad (55)$$

Hence the difference in distances covered by two particles with velocities differing by Δu is:

$$\Delta L = \frac{1}{H_0} \int_0^z \frac{dz}{(1+z)^{5/2}} (\Delta u) \quad (56)$$

where we allow Δu to depend on z .

In the context of our quantum-gravity-induced refractive-index phenomenon (36), we are confronted with just such a situation. Consider in that context two photons travelling with velocities very close to c , whose present-day energies are E_1 and E_2 . At earlier epochs, their energies would have been blueshifted by a common factor $1+z$. Defining $\Delta E_0 \equiv E_1 - E_2$, we infer from (37) that $\Delta u = (\Delta E_0 \cdot (1+z))/M$. Inserting this into (56), we find an induced difference in the arrival time of the two photons given by

$$\Delta t = \frac{\Delta L}{c} \simeq \frac{2}{H_0} \left[1 - \frac{1}{(1+z)^{1/2}}\right] \frac{\Delta E_0}{M}. \quad (57)$$

The expression (57) describes the corrections to the refractive index effect (38) due to the cosmological expansion. For small $z \ll 1$, the general expression (57) yields $\Delta t \simeq (z \cdot \Delta E_0)/(H_0 \cdot M)$, which agrees with the simple expectation $\Delta t \simeq (r \cdot \Delta E_0)/(c \cdot M)$ for a nearby source at distance $r = c(t_0 - t) \simeq cz/H_0 + \dots$. There would be similar cosmological corrections to the stochastic effect (40), also given by an expression of the form (57), but with $\Delta E_0 \rightarrow E$, $M \rightarrow \Lambda$, where E is a typical energy scale in a single channel.

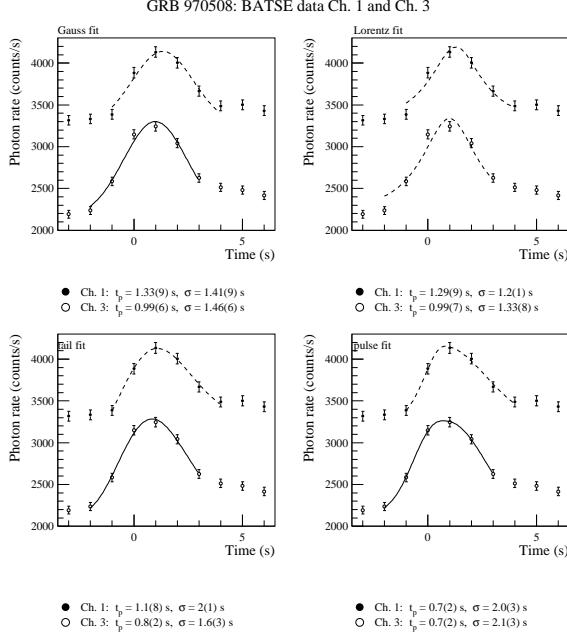


Figure 3. *Time distribution of the number of photons observed by BATSE in Channels 1 and 3 for GRB 970508, compared with the following fitting functions [27]: (a) Gaussian, (b) Lorentzian, (c) ‘tail’ function, and (d) ‘pulse’ function. We list below each panel the positions t_p and widths σ_p (with statistical errors) found for each peak in each fit. We recall that the BATSE data are binned in periods of 1.024 s.*

3.6. Sample Analysis of Data from GRBs

We presented in [27] a detailed analysis of the astrophysical data for a sample of Gamma Ray Bursters (GRB) whose redshifts z are known (see Fig. 3 for the data of a typical burst: GRB 970508). We looked (without success) for a correlation with the redshift, calculating a regression measure (see Fig. 4) for the effect (57) and its stochastic counterpart (40). Specifically, we looked for linear dependences of the ‘observed’ $\Delta t/\Delta E_0$ and the spread $\Delta\sigma/E$ on $\tilde{z} \equiv 2 \cdot [1 - (1/(1+z))^{1/2}] \simeq z - (3/4)z^2 + \dots$. We determined limits on the quantum gravity scales M and Λ by constraining the possible magnitudes of the slopes in linear-regression analyses of the differences between the arrival times and widths of pulses in different energy ranges from five GRBs with measured redshifts, as functions of \tilde{z} (57). Using the current value for the Hubble expansion parameter, $H_0 = 100 \cdot h_0$ km/s/Mpc, where $0.6 < h_0 < 0.8$, we obtained the following limits [27]

$$M \gtrsim 10^{15} \text{ GeV}, \quad \Lambda \gtrsim 2 \times 10^{15} \text{ GeV} \quad (58)$$

on the possible quantum-gravity effects.

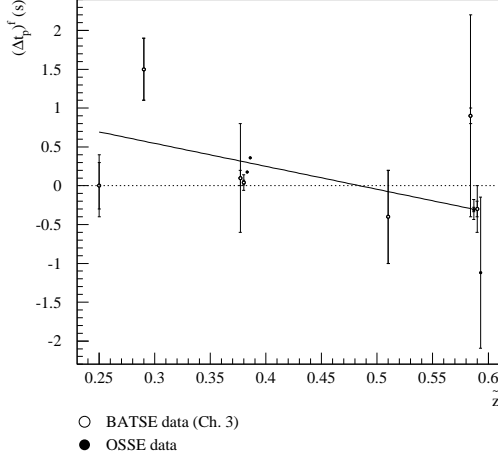


Figure 4. Values of the shifts $(\Delta t_p)^f$ in the timings of the peaks fitted for each GRB studied using BATSE and OSSE data [27], plotted versus $\tilde{z} = 1 - (1+z)^{-1/2}$, where z is the redshift. The indicated errors are the statistical errors in the ‘pulse’ fits provided by the fitting routine, combined with systematic error estimates obtained by comparing the results obtained using the ‘tail’ fitting function. The values obtained by comparing OSSE with BATSE Channel 3 data have been rescaled by the factor $(E_{min}^{BATSE \text{ Ch. 3}} - E_{max}^{BATSE \text{ Ch. 1}})/(E_{min}^{OSSE} - E_{max}^{BATSE \text{ Ch. 3}})$, so as to make them directly comparable with the comparisons of BATSE Channels 1 and 3. The solid line is the best linear fit.

4. Other Phenomenological Issues

4.1. Extension to Massive Relativistic Particles

So far, we have concentrated our attention on massless particles, namely photons, because of the very interesting experimental/observational possibilities they provide. It should however be clear, that the metric perturbation (20) produced by our D -brane recoil model, which implies a breakdown of Lorentz invariance, alters the Einstein dispersion relation for massive particles too. One expects, on general grounds, that the photon dispersion relation,

$$\omega^2 - k^2 + 2\bar{u}k\omega = 0 \quad (59)$$

will become in the case of massive particles,

$$\omega^2 - k^2 + 2\bar{u}k\omega - m_o^2 = 0, \quad (60)$$

leading to the modified Einstein relation,

$$\omega^2 \approx (k^2 + m_o^2) \left[1 - \frac{k^2}{\sqrt{k^2 + m_o^2}} \frac{1}{M} \right]^2 \quad (61)$$

to leading order in $\mathcal{O}\left(\frac{k}{M}\right)$. It is useful to recast (61) in the more familiar notation:

$$E^2 = (p^2 + m_o^2) \left[1 - \left(\frac{1}{\sqrt{1 + \frac{m_o^2}{p^2}}} \right) \frac{p}{M} \right]^2, \quad (62)$$

which in the non-relativistic limit yields:

$$E = m_o + \frac{p^2}{2m_o} - \frac{p^2}{M} + \dots \mathcal{O}\left(\left(\frac{p}{M}\right)^2\right), \quad (63)$$

Thus, an effective rest mass

$$(m_{eff})_o \approx m_o \left(1 + \frac{2m_o}{M} \right). \quad (64)$$

appears in the non-relativistic kinetic energy: $E_{kin} = \frac{p^2}{2m_{eff}}$. Alternatively, one may use (62) to recast the T-shirt formula $E = mc^2$ in the form

$$E \approx mc^2 \left[1 - \frac{m}{M} \left(\frac{\overline{u}^2}{c^2} \right) \right], \quad (65)$$

where $m \equiv \frac{m_o}{\sqrt{1 - \overline{u}^2/c^2}}$, as usual.

We see again that although quantum gravitational fluctuations in spacetime, as modelled by D -brane quantum recoil, may lead to a spontaneous breakdown of Lorentz symmetry, the resulting corrections to the standard Einstein relations are very small, so that conventional Special Relativity is still a good approximation to the world.

Before closing this subsection, we recall that another possible probe of quantum-gravitational effects on massive particles is offered by tests of quantum mechanics in the neutral kaon system. A parametrization of possible deviations from the Schrödinger equation has been given [3], assuming energy and probability conservation, in terms of quantities α, β, γ that must obey the conditions

$$\alpha, \gamma > 0, \quad \alpha\gamma > \beta^2 \quad (66)$$

stemming from the positivity of the density matrix ρ . These parameters induce quantum decoherence and violate CPT [35]. Experimental data on neutral kaon decays so far agree perfectly with conventional quantum mechanics, imposing only the following upper limits [36]:

$$\alpha < 4.0 \times 10^{-17} \text{GeV}, \quad \beta < 2.3 \times 10^{-19} \text{GeV}, \quad \gamma < 3.7 \times 10^{-21} \text{GeV} \quad (67)$$

We cannot help being impressed that these bounds are in the ballpark of m_K^2/M_P , which is the maximum magnitude that we could expect any such effect to have.

This and the example of photon propagation give hope that experiments may be able to probe physics close to the Planck scale, if its effects are suppressed by only one power of $M_P \simeq 10^{19}$ GeV. One should not exclude the possibility of being able to test some of the speculative ideas about quantum gravity reviewed in this article. Indeed, *if* the analysis of photon propagation can be extended to energetic neutrinos, and *if* GRBs emit ~ 10 s ν pulses, one could be sensitive to mass scales as large as 10^{28} GeV!

4.2. *D-particle Recoil and Time-Dependent Vacuum Energy*

We would now like to describe briefly another interesting consequence [37] of our recoil formalism, namely the generation of a time-dependent vacuum energy. We recall that the possibility of non-zero vacuum energy has recently received dramatic support from an unexpected source, namely observations of high-redshift supernovae [38]. These indicate that the large-scale geometry of the Universe is not that of a critical matter-dominated cosmology, and that its expansion may even be accelerating. The supernova data are consistent with $\Omega_\Lambda \sim 0.7$, if the Universe is indeed close to critical as suggested by inflation. The supernova and other data are beginning to provide interesting constraints on the equation of state of this vacuum energy, which is consistent with being a cosmological constant, but some time dependence in the vacuum energy is also consistent with the data.

For the purposes of this subsection, we consider cosmology in the framework of a ‘material reference system’ (MRS) of D particles [37], including the effects of quantum recoil induced by closed-string probe particles. For reasons of supersymmetry, we expect that the the ground state of the quantum-mechanical system of closed strings and D particles should have zero energy, if recoil fluctuations of the D particles are ignored, in which case there would be no cosmological vacuum energy. However, the findings of [37], which we recapitulate briefly below, indicate that, due to the recoil process, the vacuum energy of the quantum-mechanical string/ D -particle system is non-zero at finite times, and only relaxes to zero asymptotically in target time. As discussed in detail in [12], the recoil fluctuations are world-sheet higher-genus effects, and as such may be viewed as quantum fluctuations about a supersymmetric ground state of \mathcal{D} particles and strings. The emergence of a cosmological constant may be interpreted as a supersymmetry-breaking contribution of the D -particle recoil, which however relaxes to zero asymptotically in (cosmic) time. Notice that the entire recoil formalism we adopted in [12] is a non-equilibrium decohering process, and from this point of view the emergence of time-dependent supersymmetry-breaking contributions to the ground-state energy should not be too shocking.

As mentioned earlier, the closed-string/ D -particle system can be compared with that of an ion lattice in a solid (c.f. \mathcal{D} -particles) with valence electrons (c.f. closed-string states) moving freely. Lattice ion vibrations (c.f. recoil of \mathcal{D} -particles) induce effective phonon interactions which bind the electrons resulting in BCS superconductivity for the ground state of this many-body system. In this example, the physical excitations above this ground state are quasiparticles, which are not the ordinary electrons. In the D -brane analogy, the quantum recoil induced by the

scattering of a closed-string state on a defect binds the string to the defect, creating a collective recoil excitation of the system [12]. Such excitations are related formally to higher world-sheet topologies of the string, i.e., quantum effects in target space-time, which break target-space supersymmetry and make a non-trivial contribution to the ground-state energy, as discussed below. However, due to the non-equilibrium nature of the process, the recoil ground-state contribution relaxes to zero asymptotically in target time.

As in previous sections, we concentrate on a single scattering event, namely the scattering of a single closed-string state by a single defect as the first step in a dilute-gas approximation for the D particles, which should be sufficient to describe qualitatively the leading behaviour of the vacuum energy of the Universe. We are unable at present to treat fully the more realistic case of an ensemble of defects with Planckian density, due to our limited understanding of the underlying microscopic dynamics.

We consider the very low-energy limit $\bar{u}_i \rightarrow 0$ in which the only non-vanishing components of the $D + 1$ -dimensional Ricci tensor, corresponding to the metric (11), are [20]:

$$R_{ii} \simeq \frac{-(D-1)/|\epsilon|^4}{(\frac{1}{|\epsilon|^4} - \sum_{k=1}^D |y_i|^2)^2} + \mathcal{O}(\epsilon^8) \quad (68)$$

If we view (68) as the $\mathcal{O}(\alpha')$ (where α' is the Regge slope) world-sheet β function of an appropriate stringy σ model with target-space metric perturbations, then, in this limiting case, the time (time t) decouples when $t \gg 0$, and one is left with a spatial D -dimensional manifold alone. Thanks to the symmetries of the string construction, this manifold is, moreover, maximally symmetric. Therefore one may reconstruct the metric from the Ricci tensor (68):

$$R_{ij} = \frac{1}{D} \mathcal{G}_{ij} R \quad (69)$$

where \mathcal{G}_{ij} is a diagonal metric corresponding to the line element:

$$ds^2 = \frac{|\epsilon|^{-8} \sum_{i=1}^D dy_i^2}{(\frac{1}{|\epsilon|^4} - \sum_{i=1}^D |y_i|^2)^2} \quad (70)$$

This metric describes the interior of a D -dimensional ball, which is the Euclideanized version of an anti-de-Sitter (AdS) space time. In its Minkowski version, one can easily check that the curvature corresponding to (70) is

$$R = -4D(D-1)|\epsilon|^4, \quad (71)$$

which is *constant* and *negative*. The radius of the AdS space is $b = |\epsilon|^{-2}$.

The Ricci tensor (68) cannot be a consistent string background compatible with conformal invariance to order α' if only tree-level world-sheet topologies are taken into account. However, as shown in [39], this conclusion no longer holds when one includes string-loop corrections. These induce a target-space cosmological constant, corresponding to a dilaton tadpole, which renders the backgrounds (68) consistent with the conformal-invariance conditions.

Alternatively, as discussed in [20], the cosmological vacuum energy may be obtained from an effective tree-level non-critical Liouville string with central-charge deficit

$$Q^2 = \Lambda \propto -2(\alpha')^2(D-1)(D-2)|\epsilon|^4 + \mathcal{O}(\epsilon^6) \quad (72)$$

As it stands, the resulting non-conformal stringy σ model (70) has a central charge deficit (72) which is negative for the expected real values of ϵ^2 , and hence corresponds to a subcritical string theory [21]. The Liouville mode φ required to dress [14] this deformed σ model should not be confused with the target time variable that appears in the previous Section. The σ -model kinetic term for this Liouville field φ is:

$$-Q^2 \int_{\Sigma} d^2\sigma \partial\varphi \partial\varphi \quad (73)$$

with $Q^2 < 0$ given by (72), from which we see that φ is an extra *space-like* target-space coordinate. Therefore, the $D+1$ -dimensional string description resembles a conventional Euclideanized formulation of quantum gravity.

Thus, we make a conventional Wick rotation to identify the analytic continuation: $\varphi \rightarrow i\varphi$ as a Liouville ‘physical’ time t . Equations (72) and (73) then imply that ϵ should itself be analytically continued in order to obtain a time-like Liouville theory:

$$t \sim i\epsilon^{-2} \quad (74)$$

With this identification, the spatial manifold (70) is identified as a spatial slice of a space-time with signature $(D, 1)$. In our approach, slices can be ordered in $i\varphi$ just as they would be ordered in t in a canonical approach to conventional quantum gravity. Explicit Lorentz covariance is sacrificed in any such splitting. However, in our case this should be regarded as a spontaneous breaking, in the sense that it is a property of the ground state of the closed-string/ D -brane theory. Since this ground state is spatially homogeneous and isotropic, it is *a priori* suitable for describing a Friedmann-Robertson-Walker (FRW) type of cosmology. This is normally thought to be invariant under local Lorentz transformations, but it also includes the possibility that there may be a preferred Machian frame⁸, as is the case in our approach.

The consistency of the resulting metric $G_{\mu\nu}^{ph}$, $\mu, \nu = 1, \dots, D+1$, with Einstein’s equations has non-trivial consequences. As the analysis [37] indicates, the physical Universe is of FRW type with a scale factor

$$R(t) = t^2 \quad (75)$$

This can be contrasted with the tree-level cosmological model of [21], where a linear expansion was found as $t \rightarrow \infty$. We see from (72) and (74) that the Universe (75) has a time-dependent vacuum energy $\Lambda(t)$ which relaxes to zero as:

$$\Lambda(t) = \Lambda(0)/t^2 = 1/R(t) \quad (76)$$

⁸ Such as that provided in the present Universe by the microwave background radiation.

In accordance to the standard Einstein's equation, this time-varying positive vacuum energy drives the cosmic expansion:

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{1}{3}\Lambda(t) \quad (77)$$

where the dot denotes a derivative with respect to the physical (Einstein) time t .

We emphasize that the cosmological background described above satisfies Einstein's equations, and therefore can be interpreted as a consistent $D+1$ -dimensional string background. It is *not* characterized by a cosmological *constant*, but by a *time-dependent* vacuum energy. This time dependence has an arrow [10], just like conventional FRW cosmology, which stems in this approach from the irreversibility of the world-sheet renormalization-group flow in two-dimensional field theories [25, 10]. This background may be interpreted as breaking Lorentz covariance spontaneously, via the non-trivial vacuum properties of the material reference frame of the D particles.

It should be stressed that the above effect is a very low-energy contribution, appearing when the recoil velocity \bar{u}_i is ignored. As we have seen in the above construction, the σ model with a target space (75) is not the original σ model for the metric (11), but one whose β function reproduces, in the $\bar{u}_i \rightarrow 0$ limit, the limiting Ricci tensor (68). In this respect, the cosmology described above may be related to the recoil-induced space-time (11) by a marginal deformation. The further effects on this cosmology of a non-trivial recoil velocity, as appropriate for the metric (20) studied in previous sections, remain to be studied.

4.3. D -brane Recoil and 'Large' Extra Dimensions

Before closing, we would like to describe briefly another possible application of the recoil approach to D -brane gravity. This last application is inspired by the recent suggestion [9] that the observable world is actually a $D3$ brane embedded in a higher-dimensional (bulk) space-time, so that only closed-string states in the gravitational supermultiplet can propagate in the bulk. Recoil effects like those described above appear in this case as well, since the non-rigid character of the $D3$ -brane shows up when it emits closed string states into the bulk, as illustrated in Fig. 3. This problem has been studied in [40], to which we refer the reader for further details. For the present purposes, we shall summarize only the main result on light-cone broadening due to the recoil fluctuations of the $D3$ brane. The geometry considered is depicted in Fig. 3: it consists of two parallel $D3$ branes, one of which represents our world, separated by a distance Λ . The other brane could be a parallel world in the sense of [9], or it be another copy of our world, obtained upon compactifying the bulk dimension, i.e., imposing periodic boundary conditions for the gravitons. We consider propagation of photons (massless particles) along the $D3$ brane, but, in order to capture the quantum uncertainty in the $D3$ brane position, we suppose that the photons travel at a distance ℓ_s (the string length) from the brane.

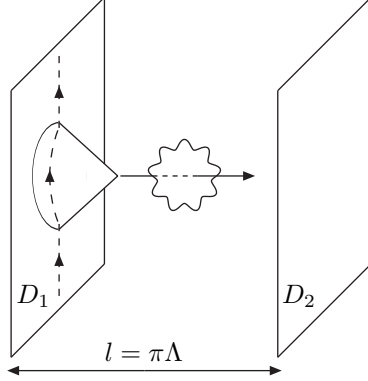


Figure 5. *Schematic representation of the recoil effect in a model with ‘large’ extra dimensions: the photon’s trajectory (dashed line) is distorted by the conical singularity in the brane that results from closed-string emission into the bulk.*

The result for the light-cone fluctuations, as discussed previously, is:

$$\frac{\Delta t}{t_p} \sim \frac{2^{n/2}}{2\pi} \left(\frac{L}{\ell_p} \right)^{1/2} \sqrt{\ln[L/\ell_p]} \left(\frac{\ell_p}{\pi\Lambda} \right)^{n/4}, \quad (78)$$

which for the plausible case $n=6$ reduces to:

$$\frac{\Delta t}{t_p} \sim \frac{4}{\pi^{5/2}} \left(\frac{L}{\Lambda} \right)^{1/2} \left(\frac{\ell_p}{\Lambda} \right) \sqrt{\ln[L/\ell_p]}. \quad (79)$$

The ratio $\ell_p/\pi\Lambda$ can be calculated in terms of the fundamental Planck scale $M_p^{(4+n)}$, as follows. Following [9], we have

$$\begin{aligned} \ell_p &= 2 \times 10^{-17} \left(\frac{1\text{TeV}}{M_p^{(4+n)}} \right) \text{cm} \\ \pi\Lambda &= 10^{30/n-17} \left(\frac{1\text{TeV}}{M_p^{(4+n)}} \right)^{1+2/n} \text{cm} \end{aligned} \quad (80)$$

whence, if $M_p^{(4+n)} = \mu \text{ TeV}$,

$$\left(\frac{\ell_p}{\pi\Lambda} \right) = 2 \times 10^{-30/n} \mu^{2/n}. \quad (81)$$

For $n=6$, this ratio is bigger than 2×10^{-5} and grows with μ as $\mu^{1/3}$, and for $\mu \sim 100$ it can be as large as 10^{-4} .

To gain an understanding whether the light-cone broadening effect might in principle be measurable, we consider some specific cases. For a GRB with redshift

$z \sim 1$, $L \sim 10^{28}\text{cm}$, and, using the above formulae, we estimate $\Lambda \sim 10^{-12}\text{cm}$, $\ell_p \sim 10^{-17}\text{cm}$ for $n = 6$, and hence

$$\Delta t \sim 10^{15}t_p, \quad (82)$$

In the extra-dimension picture the fundamental Planck time is very much larger than usually supposed in four dimensions, being of order 10^{-27} seconds, so that the light-cone broadening is 10^{-12} seconds. However, this is still far below the sensitivity of experiments measuring γ -ray bursts [5], which is in the millisecond region. It is easy to see that the effect is even smaller for $n < 6$. The above estimates have been made for $\mu = 1$: if the underlying scale is significantly higher than this, both estimates would get larger. In the case of gravity-wave interferometers, the sensitivity of the experiments is much better [8], namely of the order of 10^{-18} metres. For this case we have $L \sim 10^3\text{cm}$, and

$$\Delta t \sim 10^2 t_p \sim 10^{-25}\text{s}, \quad (83)$$

which is in principle testable at current or future gravity-wave interferometers, provided there is a controlled way to distinguish this effect from conventional noise sources⁹. However, it should be stressed that this estimate is very preliminary and much more detailed work is necessary before even tentative conclusions are reached regarding experimental tests of the effect in inteferometers. The above-mentioned sensitivity estimates could become meaningful only when a detailed description of the time of observation and the inteferometer spectrum is given.

Before closing this section, we comment briefly on the computation of the same phenomenon in extra dimensions made in the last paper of [23]. Due to the periodic boundary conditions imposed in that case, there are Kaluza-Klein modes which are resummed using the image method. The case of more than one extra dimension complicates the analysis and was not considered in detail in [23]. For one extra dimension, the light-cone broadening effect was found to scale linearly with the distance travelled by the photon, in contrast to our estimate (78). We stress therefore that this scaling depends crucially on the boundary conditions as well as the number of non-compact dimensions.

5. Conclusions

We have discussed here some possible low-energy probes of quantum gravity, concentrating on the possibility that the velocity of light might depend on its frequency, i.e., the corresponding photon energy. This idea is very speculative, and the model calculations that we have reviewed require justification and refinement. However, we feel that the suggestion is well motivated by the basic fact that gravity abhors rigid bodies, and the related intuition that the vacuum should exhibit back-reaction

⁹ In new inteferometers the length travelled by the photons may actually be higher, of order 10^5 cm, and one should probably use the optical length of the photons for such tests, which may be higher than the actual length by a factor of order $10^2 - 10^3$. We thank G. Amelino-Camelia for pointing this out.

effects and act as a non-trivial medium. We recall that these features have appeared in several approaches to quantum gravity, including the canonical approach and ideas based on extra dimensions. Therefore, we consider the motivation from fundamental physics for a frequency-dependent velocity of light, and the potential significance of any possible observation, to be sufficient to examine this possibility from a phenomenological point of view.

As could be expected, we have found no significant effect in the data available on GRBs [27], either in the possible delay times of photons of higher energies, or in the possible stochastic spreads of velocities of photons with the same energy. However, it has been established that such probes may be sensitive to scales approaching the Planck mass, if these effects are linear in the photon energy. We expect that the redshifts of many more GRBs will become known in the near future, as alerts and follow-up observations become more effective, for example after the launch of the HETE II satellite [41, 42]. Observations of higher-energy photons from GRBs would be very valuable, since they would provide a longer lever arm in the search for energy-dependent effects on photon propagation. Such higher-energy observations could be provided by future space experiments such as AMS [43] and GLAST [44].

This is not the only way in which quantum gravity might be probed: an example that we have advertized previously is provided by tests of quantum mechanics in the neutral-kaon system [3, 35, 36]. Forthcoming data from the DAΦNE accelerator may provide new opportunities for this quest. An alternative possibility might be provided by interferometric devices intended to detect gravity waves [8]. We also regard the emerging astrophysical suggestion of non-vanishing cosmological vacuum energy as a great opportunity for theoretical physics. If confirmed, this would provide a number to calculate in a complete quantum theory of gravity. We have emphasized here the possibility that this vacuum energy might not be constant, but might actually be relaxing towards zero, a possibility that may be tested by forthcoming cosmological observations. We therefore believe that the phrase ‘experimental quantum gravity’ may not be an oxymoron.

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